AM205 Quiz 1. Data fitting

Q1

What is the minimal degree of a polynomial passing through the following points

x	-1	0	1	2
y	1	0	1	4

☐ 1
☐ 2
☐ 3
☐ 4 or higher

Q2

How many polynomials of degree n pass through points $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ with distinct $x_1, \ldots, x_n$?

☐ zero
☐ one
☐ infinitely many
☐ depends on S

Q3

Suppose that $f(x)$ is an interpolating spline of degree 5, i.e. $f(x)$ is a piecewise polynomial of degree 5 in each of n segments $[x_{i-1}, x_i]$ for $i = 1, \ldots, n$ and $f(x)$ passes through points (x_i, y_i) for $i = 0, \ldots, n$. In addition, $f(x)$ is required to have 4 continuous derivatives in $[x_0, x_n]$. How many additional constraints need to be imposed on $f(x)$ to guarantee its uniqueness?

Q4

Without accounting for the rounding error, which of the following polynomials interpolates discrete data more accurately?

☐ an interpolating polynomial constructed in the monomial basis?
☐ an interpolating polynomial constructed in the Lagrange polynomial basis?
☐ both have the same accuracy

Q5

Suppose that $L_i(x),\ i = 0, \ldots, n$ represent the Lagrange polynomial basis on points $\{x_i\}$. What is the degree of $g(x) = x\big(\sum_{i=0}^{n} L_i(x)\big)$?

Q6

Which of the following are examples of nonlinear least-squares fitting problems?

☐ fitting a straight line to discrete data
☐ fitting a quadratic polynomial *to* discrete data

Q7

Interpolating polynomials are often used for function approximation. Two common choices for the set of interpolation points are the equidistant points and Chebyshev points. Which of the two will produce a lower approximation error?

☐ equidistant points
☐ Chebyshev points
☐ depends on the function

Q8

Linear least-squares fitting is equivariant to scaling if fitting to data points (x_i, y_i) and $(\lambda x_i, y_i)$ results in functions $f(x)$ and $\tilde{f}(x)$ which are related as $f(x) = \tilde{f}(x/\lambda)$ for any $\lambda > 0$. Fitting in which of the following basis functions will be equivariant to scaling?

☐ monomials: 1, x, x^2
☐ Lagrange polynomials: $(x-2)(x-3)$, $(x-1)(x-3)$, $(x-1)(x-2)$
☐ exponentials: e^{-x}, 1, e^x
☐ logarithms: 1, $\log x$, $(\log x)^2$

Q9

Suppose that the first derivative of $f(x) = \sin(x)$ is calculated at $x = 1$ using a finite difference approximation $\frac{f(x+h)-f(x)}{h}$ for an arbitrarily small $h > 0$. What is the best relative error you expect to achieve from a calculation with double precision?

☐ 1
☐ 10^{-8}
☐ 10^{-16}

Q10

How many bits are used to represent a number in the IEEE double precision arithmetic?

Q11

In the context of function approximation using a polynomial interpolant, what are the advantages of Chebyshev points over equidistant points?

☐ the degree of the polynomial grows slower with the number of points
☐ smaller error for the same number of points

☐ lower algorithmic complexity of evaluating the Lagrange polynomial

Q12

Runge's function is an example of

☐ a smooth function for which the polynomial interpolation using equidistant points leads to an exponential growth of the infinity norm error
☐ a basis function equivariant to translation
☐ a continuous function with a discontinuous first derivative

Q13

What is the maximum absolute value of a Chebyshev polynomial $T_{n+1}(x)$ in the range $[-1, 1]$?

☐ 1
☐ 2^{n+1}
☐ $1/2^{n+1}$
☐ $n+1$