AM205 Quiz 2. Numerical linear algebra

Q1

Which of the vector norm axioms are violated for the p-norm if $0 < p < 1$?

- ☐ absolute homogeneity
- ☐ triangle inequality
- ☐ positive definiteness
- ☐ none of the above

Q2

The product of two upper triangular matrices is an upper triangular matrix.

- ☐ true
- ☐ false

Q3

Consider a matrix $A \in \mathbb{R}^{n \times n}$ and vector $b \in \mathbb{R}^n$. Assume that an LU factorization of A is known. What is the complexity of solving the linear system $Ax = b$ using that LU factorization?

- ☐ $\mathcal{O}(n)$
- ☐ $\mathcal{O}(n^2)$
- ☐ $\mathcal{O}(n^3)$
- ☐ none of the above

Q4

Let L_j be an elementary elimination matrix from one step of the LU factorization algorithm for a square matrix A. Which of the following statements are correct in general for any A? The matrix L_j is

- ☐ invertible
- ☐ lower triangular
- ☐ orthogonal
- ☐ sparse
- ☐ none of the above

Q5

Suppose that a square matrix A has a Cholesky factorization $A = LL^T$, where L is a square invertible lower triangular matrix. Which of the following statements are correct in general for any L? The matrix A is

- ☐ lower triangular
- ☐ positive-definite

☐ symmetric
☐ none of the above

Q6

Which of the following factorizations of a square matrix are unique?

☐ LU
☐ QR
☐ none of the above

Q7

Suppose that F is a Householder reflector. Which of the following statements are correct in general?

☐ F is orthogonal
☐ $F^2 = I$
☐ none of the above

Q8

Suppose that Q is an orthogonal matrix and $Q = U\Sigma V^T$ is its singular value decomposition. Which of the following statements are correct in general?

☐ Σ is diagonal
☐ Σ is invertible
☐ $\|\Sigma\|_2 = 1$
☐ none of the above

Q9

Consider a matrix $A \in \mathbb{R}^{n \times n}$ and vector $b \in \mathbb{R}^n$. Which of the following factorizations, once known, reduce the complexity of solving the linear system $Ax = b$ to $\mathcal{O}(n^2)$?

☐ LU
☐ QR
☐ SVD
☐ Cholesky
☐ none of the above