AM205 Quiz 3. Numerical calculus. Solution

Q1

Newton-Cotes formulas are quadrature rules that

☒ are obtained by integrating a polynomial interpolant
☐ use Newton's method to find the quadrature weights

Q2

Consider a quadrature rule $Q(f) = \sum_{k=0}^{n} w_k f(k)$ to approximate the integral $\int_0^2 f(x)dx$. This rule uses $n+1$ function values at integer points $0, \ldots, n$. However, only the points $0, 1, 2$ belong to the integration range $[0, 2]$. Suppose that the constant weights $w_0, \ldots, w_n$ are chosen to maximize the degree of polynomials on which this quadrature is exact. This implies that the quadrature rule is exact on all polynomials of degree (choose the highest)

☐ 2
☐ 3
☒ n
☐ $n+1$

Answer: Newton-Cotes formulas using an interpolating polynomial will be exact on polynomials of degree n regardless of the integration range. The interpolating polynomial coincides with the integrand.

Q3

Gauss quadrature using $n+1$ points is exact on all polynomials of degree (choose the highest)

☐ n
☒ $2n+1$

Q4

The centered difference approximation $\frac{f(x+h)-f(x-h)}{2h}$ to $f'(x)$ is exact on all polynomials $f(x)$ of degree (choose the highest)

☐ 0
☐ 1
☒ 2
☐ 3
☐ 4

Q5

Compare two methods for solving a system of linear ODEs: forward Euler (explicit) and backward Euler (implicit). The backward Euler method

☒ has a larger stability region
☐ has a higher order of accuracy
☒ requires solving a linear system at every time step
☐ none of the above

Q6

Richardson extrapolation applies to an existing numerical method and can be used to

☒ increase its order of accuracy
☒ estimate its order of accuracy
☒ estimate its absolute error
☐ none of the above

Q7

Compare one-step and multistep methods for solving ODEs. To achieve the same order of accuracy, multistep methods require

☒ fewer function evaluations
☐ more function evaluations

Q8

Recall the θ-method for the heat equation. The method is fully explicit with $\theta = 0$ and fully implicit with $\theta = 1$. Which of the following methods are unconditionally stable?

☒ Crank-Nicolson
☐ θ-method with $\theta = 0.25$
☒ θ-method with $\theta = 0.5$
☒ θ-method with $\theta = 1$
☐ none of the above

Q9

Examples of hyperbolic equations are

☒ advection equation
☐ heat equation
☐ Poisson equation
☒ wave equation
☐ none of the above

Q10

Recall the central difference method $\frac{U_j^{n+1}-U_j^n}{\Delta t} + c\frac{U_{j+1}^n - U_{j-1}^n}{2\Delta x} = 0$ for the advection equation. Even if $c\Delta t/\Delta x$ is small, the method cannot be used in practice for the following reasons:

☐ does not satisfy the CFL condition
☒ unconditionally unstable
☐ requires two boundary conditions