AM205 Quiz 4. Optimization

Q1

Suppose that $g : \mathbb{R} \to \mathbb{R}$ is a nonlinear smooth function with a fixed point $\alpha \in \mathbb{R}$, i.e. $g(\alpha) = \alpha$. Which of the following statements are true in general?

- ☐ $g'(\alpha) = 1$
- ☐ $|g'(\alpha)| < 1$
- ☐ $|g'(\alpha)| \leq 1$
- ☐ none of the above

Q2

Suppose that a sequence x_k converges linearly to α. Define $y_k = (x_k - \alpha)^2$. Which of the following statements is true in general?

- ☐ y_k converges linearly to 0
- ☐ y_k converges superlinearly to 0

Q3

Consider a scalar equation $f(x) = 0$ with a smooth and strictly convex function $f : \mathbb{R} \to \mathbb{R}$. Which of the following methods are expected to converge **superlinearly**? Assume that the initial guess is chosen sufficiently close to a solution.

- ☐ bisection method
- ☐ Newton's method
- ☐ secant method
- ☐ none of the above

Q4

Consider a continuous function $f : \mathbb{R} \to \mathbb{R}$. Which of the following statements are true?

- ☐ if f is coercive on $\mathbb{R}$, then f has a global minimum in $\mathbb{R}$
- ☐ if f has a unique global minimum in $\mathbb{R}$, then f is coercive on $\mathbb{R}$
- ☐ none of the above

Q5

The function $f(x) = |x|$ defined on $\mathbb{R}$ is

- ☐ coercive
- ☐ convex
- ☐ strictly convex
- ☐ none of the above

Q6

The Hessian of the function $f(x, y) = x^2 + y^2$ is

- ☐ positive definite
- ☐ negative definite
- ☐ indefinite
- ☐ none of the above

Q7

To optimize a function $f : \mathbb{R}^n \to \mathbb{R}$, the BFGS algorithm relies on evaluations of

- ☐ the function f
- ☐ the gradient ∇f
- ☐ the Hessian H_f

Q8

Recall the Lagrangian function $\mathcal{L}(b, \lambda) = b^T b + \lambda^T (Ab - y)$ corresponding to an under-determined linear least squares problem. Assume that $A \in \mathbb{R}^{m \times n}$ has full rank and $m \leq n$. Suppose that this function is minimized using Newton's method with a zero initial guess $b_0 = 0$ and $\lambda_0 = 0$. How many iterations would Newton's method need to satisfy $\|\nabla \mathcal{L}\|_2 < 10^{-5}$?

- ☐ one
- ☐ depends on $\|A\|_2$
- ☐ depends on $\|A\|_2$ and $\|y\|_2$